**Side-by-Side Comparison of Current and Proposed Endorsement Competencies with Alignment to CCSS-M & STEM Integration**

**Mathematics (5-12) Endorsement**

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| **Current Competencies** | **Proposed Competencies** |
| 1.0 Mathematical Problem Solving:  Teachers know, understand, and apply the process of mathematical problem solving. 1.A Apply and adapt a variety of appropriate strategies to solve problems. 1.B Solve problems that arise in mathematics and those involving mathematics in other contexts. 1.C Build new mathematical knowledge through problem solving. 1.D Monitor and reflect on the process of mathematical problem solving. | 1.0 Standards for Mathematical Practices  Demonstrate ability to embed CCSS-M Mathematical Practices in the instructional process to deepen conceptual understanding.  1.A Make sense of problems and persevere in solving them.  1.B Reason abstractly and quantitatively.  1.C Construct viable arguments and critique the reasoning of others.  1.D Model with mathematics.  1.E Use appropriate tools strategically.  1.F Attend to precision.  1.G Look for and make use of structure.  1.H Look for and express regularity in repeated reasoning. |
| 7.0 Number and Operation:  Teachers demonstrate computational proficiency, including a conceptual understanding of numbers, ways of representing number, relationships among number and number systems, and meanings of operations. 7.A Analyze and explain the mathematics that underlies the procedures involving operations with whole numbers, integers, rational, real, and complex numbers. 7.B Recognize the meaning and use of place value in representing whole numbers and finite decimals, comparing and ordering numbers, and understanding the relative magnitude of numbers. 7.C Demonstrate proficiency in multi-digit computation using algorithms, mental mathematics, and computational estimation. 7.D Provide equivalent representations of fractions, decimals, and percents. 7.E Create, solve, and use proportions. 7.F Apply the fundamental ideas of number theory. 7.G Make sense of large and small numbers and use scientific notation. 7.H Compare and contrast properties of numbers and number systems. 7.I Represent, use, and apply properties of complex numbers. 7.J Recognize matrices and vectors as systems that have some of the properties of the real number system. 7.K Demonstrate knowledge of the historical development of number and number systems, including contributions from many cultures. | 2.0 Number and Quantity  Candidates demonstrate a conceptual understanding of and procedural facility with operations and number systems.  2.A Understand the structure, properties, characteristics of, and relationships between number systems including whole numbers, integers, rational, real, and complex numbers.  2.B Understand arithmetic operations of different number systems and their properties.  2.C Understand vectors and matrices.  2.C.1 Represent and model with vector quantities and matrices.  2.C.2 Perform operations on vectors and matrices.  2.C.3 Understand the properties of operations using vectors and matrices. |
| 8.0 Algebra Teachers understand relationships among quantities including functions, ways of representing mathematical relationships, and the analysis of change. 8.A Explore, predict, analyze, and represent patterns, relations, and functions. 8.B Apply fundamental ideas of linear algebra. 8.C Investigate equality, equations, inequalities, and proportional relationships. 8.D Use mathematical models including technological tools to represent and understand quantitative relationships. 8.E Analyze the concept of change in various contexts. 8.F Demonstrate knowledge of the historical development of algebra, including contributions from many cultures. | 3.0 Algebra and Functions  Candidates demonstrate a conceptual understanding of and procedural facility with algebra concepts emphasizing functions.  3.A Use functional notation and interpret functions as they arise in different mathematical contexts.  3.B Understand operations on algebraic expressions and functions (e.g., polynomials, rationals, and roots).  3.C Understand and prove properties of algebraic systems.  3.D Create mathematical models using algebraic expressions.  3.E Solve algebraic equations and inequalities with one or more variables.  3.F Write equations and inequalities in equivalent forms (e.g., graphs, tables, and algebraic expressions) to gain a more complete understanding to solve problems.  3.G Explain the connection among equivalent forms and their algebraic purposes.  3.H Conceptually understand functions and build models of relationships between two quantities.  3.I Analyze and model functions (e.g., linear quadratic, exponential, and trigonometric). |
| 9.0 Geometry:  Teachers use spatial visualization and geometric modeling to explore and analyze geometric figures, structures, and their properties. 9.A Demonstrate knowledge of core concepts and principles of Euclidean and non- Euclidean geometries in two and three dimensions from both formal and informal perspectives. 9.B Exhibit knowledge of the role of axiomatic systems and proofs in geometry. 9.C Analyze characteristics and relationships of geometric objects and figures. 9.D Build and manipulate representations of two- and three- dimensional objects using concrete models, drawings, and dynamic geometry software, and perceive an object from different perspectives. 9.E Specify locations and describe spatial relationships using coordinate geometry, vectors, and other representational systems. 9.F Apply transformations and use symmetry, similarity, and congruence in mathematical situations. 9.G Demonstrate knowledge of the historical development of Euclidean and non- Euclidean geometries, including contributions from many cultures.  11.0 Measurement:  Teachers apply and use measurement concepts and tools. 11.A Recognize the common representations and uses of measurement and choose tools and units for measuring. 11.B Identify the attributes to be measured and apply appropriate techniques, tools, and formulas to determine measurements and their application in a variety of contexts. 11.C Use estimation as a way of understanding measurement units and processes. 11.D Demonstrate knowledge of the historical development of measurement including contributions from many cultures. | 4.0 Geometry and Measurement  Candidates prove and understand geometric theorems and transformations as they apply to congruence, similarity, lines, triangles, trigonometry, circles, and measurements. In addition, the following content should be included; the relationship between geometric figures and the Cartesian coordinate system.  4.A Understand congruence in terms of rigid motions and prove geometric theorems.  4.B Apply transformations and use similarity and congruence in mathematical situations.  4.C Perform geometric constructions physically and/or with technology.  4.D Define trigonometric ratios and solve problems involving right triangles and general triangles.  4.E Derive the Pythagorean Theorem and apply it to problem solving situations.  4.F Identify and describe relationships among angles, radii, and chords.  4.G Derive formulas for arc lengths and areas of sectors of circles.  4.H Translate between the geometric description and the equation for a conic section.  4.I Use coordinates to prove geometric theorems algebraically.  4.J Derive area, surface area, and volume formulas and use them to solve problems.  4.K Solve real life and mathematical problems involving angle measures and/or polygons.  4.L Visualize and describe two-dimensional figures and three-dimensional objects as well as the relationships among them.  4.M Apply geometric concepts to model real world situations. |
| 10.0 Data Analysis, Statistics, and Probability:  Teachers demonstrate an understanding of concepts and practices related to data analysis, statistics, and probability. 10.A Design investigations, collect data, use a variety of ways to display data, interpret data representations and draw and represent conclusions that may include bivariate data and geometric probability. 10.B Use appropriate methods such as random sampling or random assignment of treatments to estimate population characteristics, test conjectured relationships among variables, and analyze data. 10.C Use appropriate statistical methods and technological tools to describe shape and analyze spread and center. 10.D Use statistical inference to draw conclusions from data. 10.E Identify misuses of statistics and invalid conclusions from probability. 10.F Draw conclusions involving uncertainty by using hands-on and technology-based simulation for estimating probabilities and gathering data to make inferences and decisions. 10.G Demonstrate knowledge of the historical development of statistics and probability, including contributions from many cultures. | 5.0 Statistics and Probability  Candidates demonstrate a conceptual understanding of and procedural facility with statistics and probability.  5.A Summarize, represent, and interpret single variable and multivariate data that can be categorical or continuous.  5.B Represent and interpret linear regression models.  5.C Apply statistical concepts and representations to model real world situations.  5.D Use appropriate technology to collect, represent, and analyze data.  5.E Make inferences, collect data from random experiments, and justify conclusions.  5.E.1 Conduct random sampling.  5.E.2 Evaluate a statistical experiment with respect to the assumption of randomization.  5.E.3 Make and justify conclusions about inferences from appropriate statistical analysis of data.  5.F Understand and apply the principles of probability (complementary events, mutually exclusive events, independent, and dependent events) to compute probabilities of compound events.  5.G Apply probability concepts to model real world situations.  5.H Use probability to make and inform decisions.  5.H.1 Calculate expected values.  5.H.2 Make inferences from binomial probability distributions (e.g., sampling populations from a binary sample space). |
| N/A | 6.0 Ratios and Proportional Relationships  Candidates recognize, analyze, and represent equivalent ratios, rates, and proportional relationships and use them to solve problems. |
| 6.0 Technology:  Teachers embrace technology as an essential tool for teaching and learning mathematics. 6.A Use knowledge of mathematics to select and use appropriate technological tools, such as but not limited to, spreadsheets, dynamic graphing tools, computer algebra systems, dynamic statistical packages, graphing calculators, data-collection devices, and presentation software. 6.B Understand the appropriate use of technology to experiment, visualize, and make/explore conjectures. | 7.0 Modeling and Technology  Candidates will be able to connect mathematics with real life problems through the use of mathematical modeling and technology.  7.A Construct mathematical models in the content strands (e.g., look at a real life situation and transpose it into a mathematical problem, solve the problem, and interpret the solution in real life.)  7.B Use the appropriate technology available.  7.B.1 Explore conjectures, visualize, and analyze the mathematics.  7.B.2 Develop concepts and apply them to a context. |
| 12.0 Calculus:  Teachers demonstrate a conceptual understanding of and procedural facility with basic calculus concepts. 12.A Demonstrate a conceptual understanding of limit, continuity, differentiation, and integration and a thorough background in the techniques and application of the calculus. 12.B Apply concepts of function, geometry, and trigonometry in solving problems involving calculus. 12.C Use the concepts of calculus and mathematical modeling to represent and solve problems taken from real-world contexts. 12.D Use technological tools to explore and represent fundamental concepts of calculus. 12.E Demonstrate knowledge of the historical development of calculus, including contributions from many cultures. | 8.0 Calculus  Candidates demonstrate a conceptual understanding of and procedural facility with calculus concepts.  8.A Demonstrate a conceptual understanding of limit, continuity, differentiation, and integration and have a thorough background in the techniques and application of the calculus.  8.B Apply concepts of function, geometry, and trigonometry in solving problems involving calculus.  8.C Use the concepts of calculus and mathematical modeling to represent and solve problems taken from real world contexts. |
| 13.0 Discrete Mathematics:  Teachers apply the fundamental ideas of discrete mathematics in the formulation and solution of problems. 13.A Demonstrate knowledge of basic elements of discrete mathematics such as graph theory, recurrence relations, finite difference approaches, linear programming, and combinatorics. 13.B Apply the fundamental ideas of discrete mathematics in the formulation and solution of problems arising from real-world situations. 13.C Use technological tools to solve problems involving the use of discrete structures and the application of algorithms. 13.D Demonstrate knowledge of the historical development of discrete mathematics, including contributions from many cultures. | 9.0 Discrete Mathematics  Candidates apply the fundamental ideas of discrete mathematics in the formulation and solution of problems.  9.A Develop the general techniques of mathematical proof including direct proofs, proof by contradiction, contrapositive proof, and proof by induction.  9.B Demonstrate knowledge of basic elements of discrete mathematics such as graph theory, recurrence relations, finite difference approaches, and combinatorics.  9.C Apply the fundamental ideas of discrete mathematics in the formulation and solution of problems arising from real world situations. |
| 14.0 Instructional Methodology:  Teachers possess a deep understanding of how students learn mathematics and of the pedagogical knowledge specific to mathematics teaching and learning. 14.A Select, use, and determine suitability of the wide variety of available mathematics curricula and teaching materials for all students including those with special needs such as the gifted, challenged and speakers of other languages. 14.B Select and use appropriate concrete materials for learning mathematics. 14.C Use multiple strategies, including listening to and understanding the ways students think about mathematics, to assess students’ mathematical knowledge and provide appropriate interventions. 14.D Plan lessons, units and courses that address Washington Essential Academic Learning Requirements (EALRs), Grade-Level Expectations (GLEs), and WASL item specifications. 14.E Demonstrate knowledge of research results in the teaching and learning of mathematics, and use print and online resources of professional mathematics organizations. 14.F Use knowledge of different types of instructional strategies, including differentiation, in planning mathematics lessons. 14.G Demonstrate ability to present mathematical concepts using multiple representations (e.g., numerical, graphical, analytical, contextual) to address the multiple learning styles of students 14.H Demonstrate the ability to use student discourse to lead classes in mathematical problem solving and develop in-depth conceptual understanding to help students develop and test generalizations. 14.I Develop lessons that use technology’s potential for building understanding of mathematical concepts and developing important mathematical ideas. 14.J Demonstrate a positive impact on student learning of mathematics. 14.K Engage in culturally responsive teaching of mathematics. 14.L Assure and support equity in the classroom, with equal access to and continued study of mathematics for all students.  2.0 Reasoning and Proof:  Teachers reason, construct, and evaluate mathematical arguments and interpret and compare mathematical information from a variety of sources. 2.A Recognize reasoning and proof as fundamental aspects of mathematics. 2.B Make and investigate mathematical conjectures. 2.C Develop and evaluate mathematical arguments and proofs. 2.D Select and use various types of reasoning and methods of proof. 3.0 Mathematical Communication:  Teachers communicate their mathematical thinking orally and in writing, using appropriate mathematical language and notation to clearly and effectively express or present ideas and information. 3.A Systematically gather mathematical information for a given purpose and clearly communicate their findings to peers, faculty, and others. 3.B Use the language of mathematics to express ideas precisely. 3.C Use communication as a means of clarifying and organizing one’s own mathematical thinking. 3.D Analyze and evaluate the mathematical thinking and strategies of others. 4.0 Mathematical Connections:  Teachers recognize, use, and make connections between and among mathematical ideas and in contexts outside mathematics to build mathematical understanding. 4.A Recognize and use connections among mathematical ideas. 4.B Recognize and apply mathematical ideas to other subject areas and to real-world situations. 4.C Demonstrate how mathematical ideas interconnect and build on one another to produce a coherent whole. 5.0 Mathematical Representation:  Teachers use varied representations (words, pictures, data representations) of mathematical ideas to support and deepen mathematical understanding. 5.A Use multiple representations to model and interpret physical, social, and mathematical phenomena. 5.B Create and use representations to organize, record, and communicate mathematical ideas. 5.C Select, apply, and translate among mathematical representations to solve problems. | 10.0 Mathematics Instructional Methodology  Candidates possess a deep understanding of how students learn mathematics and of the pedagogical knowledge specific to mathematics teaching and learning.  10.A Select, use, and determine suitability of the available mathematics curricula, teaching materials, and other resources including manipulatives for the learning of mathematics for all students.  10.B Demonstrate ability to present mathematical concepts using multiple representations (e.g., numerical, graphical, analytical, and contextual).  10.C Demonstrate the ability to guide student discourse in mathematical problem solving, argumentation (creation and critiquing), literacy, and in-depth conceptual understanding.  10.D Demonstrate knowledge of learning progressions, including conceptual and procedural milestones and common misconceptions, within each content domain and connections to instruction.  10.D.1 Demonstrate knowledge of major, supporting, and additional clusters for each grade level.  10.D.2 Demonstrate an understanding of the concept of mathematical rigor including conceptual understanding, procedural skill and fluency, and application.  10.D.3 Demonstrate an understanding of coherent connections within clusters at a grade level and the progression from grade level to grade level that builds on previous learning.  10.E Engage in developmentally and culturally responsive teaching of mathematics that minimizes power and status issues, nurtures a positive mathematics disposition, and utilizes students’ cultural funds of knowledge and experiences as resources for lessons. |