**Side-by-Side Comparison of Current and Proposed Endorsement Competencies with Alignment to CCSS-M & STEM Integration**

**Middle Level Mathematics (4-9) Endorsement**

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| **Current Competencies** | **Draft Competencies** |
| 1.0 Mathematical Problem Solving Middle level teacher candidates know, understand, and apply the process of mathematical problem solving in each mathematical content area. As a result, candidates:  1.A Apply and adapt a variety of appropriate strategies to solve problems.  1.B Solve problems that arise in mathematics and those involving mathematics in other contexts.  1.C Build new mathematical knowledge through problem solving.  1.D Monitor and reflect on the process of mathematical problem solving and evaluate reasonableness of solution. | 1.0 Standards for Mathematical Practices  Demonstrate ability to embed CCSS-M Mathematical Practices in the instructional process to deepen conceptual understanding.  1.A Make sense of problems and persevere in solving them.  1.B Reason abstractly and quantitatively.  1.C Construct viable arguments and critique the reasoning of others.  1.D Model with mathematics.  1.E Use appropriate tools strategically.  1.F Attend to precision.  1.G Look for and make use of structure.  1.H Look for and express regularity in repeated reasoning. |
| 6.0 Number and Operation Middle level teacher candidates demonstrate computational proficiency using various strategies, including a conceptual understanding of numbers, ways of representing number (numbers, tables, graphs, models, words, and symbols), relationships among number and number systems, and meanings of operations with all real numbers. As a result, candidates:  6.A Analyze and explain the mathematics that underlies the procedures involving operations with real and complex numbers.  6.B Recognize the meaning and use of place value in representing whole numbers and finite decimals, comparing and ordering numbers, and understanding the relative magnitude of numbers  6.C Demonstrate proficiency in real number computation using multiple algorithms, mental mathematics, and computational estimation.  6.D Demonstrate understanding, representation, and use of fractions, decimals, and percents.  6.E Demonstrate understanding of the meaning of operations on fractions, decimals, and percents.  6.F Provide equivalent representations of fractions, decimals, and percents.  6.G Demonstrate understanding and use of proportional reasoning (direct and inverse variation) to solve problems.  6.H Demonstrate understanding of rates and ratios, and use them to solve problems.  6.I Demonstrate understanding of the fundamental ideas of number theory (e.g. divisibility, factoring, multiples, prime factorization, prime and composite)  6.J Demonstrate understanding of and use exponential and scientific notations.  6.K Compare properties of number systems.  6.L Represent, use, and apply properties of complex numbers.  6.M Recognize matrices and vectors as systems that have some of the properties of the real number system.  6.N Demonstrate understanding of the historical development of number and number systems, including contributions from many cultures. | 2.0 Number and Quantity  Candidates demonstrate a conceptual understanding of and procedural facility with operations and number systems.  2.A Understand the structure, properties, characteristics of, and relationships between different number systems including whole numbers, integers, rational, real, and complex numbers.  2.B Understand arithmetic operations of different number systems and their properties (integers, rational, and irrational numbers).  2.C Understand the progression of learning that begins with the base-ten number system and operations thereof, builds into understanding of and operations with fractions and rational numbers, and extends to understanding of and operations with real numbers. |
| 7.0 Algebra Middle level teacher candidates understand relationships among quantities, functions, ways of representing mathematical relationships (numbers, tables, graphs, models, words, and symbols), and the analysis of change. As a result, candidates:  7.A Identify, explore, analyze, predict, and represent patterns, relations, and functions.  7.B Apply techniques of algebra to linear, quadratic, and exponential functions.  7.C Demonstrate understanding of the relationships of equations and inequalities; including proportional reasoning.  7.D Use mathematical models including technological tools to represent and demonstrate understanding of quantitative relationships and functions.  7.E Analyze the concept of change in various contexts.  7.F Demonstrate understanding of the historical development of algebra, including contributions from many cultures. | 3.0 Algebra and Functions  Candidates demonstrate a conceptual understanding of and procedural facility with algebra concepts emphasizing functions.  3.A Solve and graphically represent real life and mathematical problems using numerical and algebraic expressions, equations, inequalities, and systems of equations and inequalities.  3.B Understand the connections between proportional relationships, lines, and linear equations and use them to solve real world and mathematical problems.  3.C Use functional notation and interpret expressions for functions as they arise in terms of the situation they model (e.g., linear, quadratic, simple rational, and exponential).  3.D Understand operations on algebraic expressions and functions (e.g., polynomials, rationals, and roots).  3.E Apply arithmetic properties to algebraic expressions and equations.  3.F Write equations and inequalities in equivalent forms.  3.G Analyze and model functions.  3.H Explain the interrelationship between the various representations of a function (e.g., graphs, tables, algebraic expressions, concrete models, and contexts). |
| 8.0 Geometry Middle level teacher candidates use spatial visualization and geometric modeling to explore and analyze geometric figures, structures, their properties, and ways of representing mathematical relationships (numbers, tables, graphs, models, words, and symbols). As a result, candidates:  8.A Demonstrate understanding of core concepts and principles of Euclidean and non-Euclidean geometries in two and three dimensions from both formal and informal perspectives.  8.B Demonstrate understanding of the role of axiomatic systems and proofs in geometry.  8.C Analyze characteristics and relationships of geometric objects and figures.  8.D Build and manipulate representations of two- and three- dimensional objects using concrete models, drawings, and dynamic geometry software, and perceive an object from different perspectives.  8.E Specify locations and describe spatial relationships using coordinate geometry, vectors, and other representational systems.  8.F Apply transformations and use symmetry, similarity, and congruence in mathematical situations.  8.G Demonstrate understanding of the historical development of Euclidean and non-Euclidean geometries, including contributions from many cultures. | 4.0 Geometry and Measurement  Candidates demonstrate a conceptual understanding of geometric properties and relationships as they apply to congruence, similarity, geometric figures, and the Cartesian Coordinate System.  4.A Understand congruence in terms of rigid motion.  4.B Prove theorems involving triangle congruency and similarity.  4.C Apply transformations and use similarity and congruence in mathematical situations.  4.D Understand and perform geometric constructions physically and/or with technology.  4.E Understand the Pythagorean Theorem and apply it to problem solving situations.  4.F Solve real life and mathematical problems involving lines, angle measure, area, surface area, and volume.  4.G Classify, visualize, and describe two-dimensional figures and three-dimensional objects as well as the relationship among them.  4.H Apply geometric concepts to model real world situations. |
| 9.0 Data Analysis, Statistics, and Probability Middle level teacher candidates demonstrate an understanding of concepts and practices related to data analysis, statistics, probability, and ways of representing mathematical relationships (numbers, tables, graphs, models, words, and symbols). As a result, candidates:  9.A Design investigations, collect data, use a variety of ways to display data, interpret data representations and draw and represent conclusions that may include bi-variate data and geometric probability.  9.B Use appropriate methods such as random sampling or random assignment of treatments to estimate population characteristics, test conjectured relationships among variables, and analyze data.  9.C Use appropriate statistical methods and technological tools to describe shape and analyze spread and center.  9.D Apply basic statistics and statistical interpretation to the analysis of data.  9.E Use statistical inference to draw conclusions from data.  9.F Identify misuses of statistics and invalid conclusions from probability.  9.G Draw conclusions involving uncertainty by using hands-on and technology-based simulation for estimating probabilities and gathering data to make inferences and decisions.  9.H Demonstrate understanding of the historical development of statistics and probability, including contributions from many cultures. | 5.0 Statistics and Probability  Candidates demonstrate conceptual understanding and procedural facility of statistics and probability.  5.A Use appropriate measures of central tendency and distributions to summarize, represent, and interpret categorical and quantitative data.  5.B Understand and evaluate random processes underlying statistical experiments and use random sampling to make inferences about whole populations.  5.C Understand and use the rules of probability to make predictions, evaluate decisions, and solve problems.  5.D Apply probability concepts to model real world situations. |
| 10.0 Measurement: Middle level teacher candidates apply and use measurement concepts and tools and ways of representing mathematical relationships (numbers, tables, graphs, models, words, and symbols). As a result, candidates:  10.A Demonstrate understanding of the common representations and uses of measurement and choose tools and units for measuring length, surface area, volume, mass, weight, angle, elapsed time, rate, and temperature.  10.B Identify the attributes to be measured and apply appropriate techniques, tools, and formulas to determine measurements and their application in a variety of contexts.  10.C Demonstrate an understanding of the distinction between precision and accuracy of measurements.  10.D Use estimation as a way of understanding measurement units and processes.  10.E Demonstrate understanding of international (metric) and customary (U.S.) units of measure and apply unit conversions within each measurement system.  10.F Demonstrate understanding of the historical development of measurement including contributions from many cultures. | 6.0 Ratios and Proportional Relationships  Candidates demonstrate conceptual understanding and procedural fluency in analyzing proportional relationships and solving real world mathematical problems.  6.A Describe and determine additive versus multiplicative perspectives.  6.B Reason and compute with ratios and the constant of proportionality (unit rate) to solve real world and mathematical problems.  6.C Recognize, describe, and represent equivalent ratios, rates, and proportional relationships.  6.D Represent and analyze proportional relationships using tables, graphs, equations, diagrams, concrete and mathematical models, and verbal descriptions of proportional relationships.  6.E Compute the constant of proportionality (unit rate) associated with rational numbers.  6.F Recognize and connect proportional relationships to geometry, measurement, statistics, probability, and function.  6.G Use ratio reasoning to convert measurement units.  6.H Apply ratio and proportion concepts to model real world situations. |
| 13.0 Integrating Technology with Mathematics Middle level teacher candidates embrace technology as an essential tool for teaching and learning mathematics. As a result, candidates:  13.A Understand the appropriate use of technology to experiment, visualize, and make/explore conjectures.  13.B Demonstrate an ability to use instructional support strategies to promote academic achievement for those students with significant gaps in their mathematical knowledge.  13.C Use technological tools, including but not limited to computer technology, to access resources, collect and process data, and facilitate learning.  13.D Use appropriate technology to help students acquire concepts and skills. | 7.0 Modeling and Technology  Candidates will be able to connect mathematics with real life problems through the use of mathematical modeling and technology.  7.A Construct mathematical models in the content strands (e.g., look at a real life situation and transpose it into a mathematical problem, solve the problem, and interpret the solution in real life.)  7.B Use the appropriate technology available.  7.B.1 Explore conjectures, visualize, and analyze the mathematics.  7.B.2 Develop concepts and apply them to a context. |
| 2.0 Reasoning and Proof Middle level teacher candidates reason, construct, evaluate and justify mathematical arguments and interpret and compare mathematical information from a variety of sources in each mathematical content area. As a result, candidates:  2.A Recognize reasoning and proof as fundamental aspects of mathematics.  2.B Make and investigate mathematical conjectures.  2.C Develop and evaluate mathematical arguments and proofs.  2.D Select and use various types of reasoning (e.g., inductive, deductive, proportional, spatial) and methods of proof.  3.0 Mathematical Communication Middle level teacher candidates communicate their mathematical thinking orally and in writing, using appropriate mathematical language and notation to clearly and effectively express or present ideas and information in each mathematical content area. As a result, candidates:  3.A Systematically gather mathematical information for a given purpose and clearly communicate their findings to peers, faculty, and others.  3.B Use the language of mathematics to express ideas precisely.  3.C Use communication as a means of clarifying and organizing one’s own mathematical thinking.  3.D Engage in inquiry, including mathematical discourse, to develop concepts and identify relationships from their observations, data, and inferences.  3.E Analyze and evaluate the mathematical thinking and strategies of others.  4.0 Mathematical Connections Middle level teacher candidates recognize, use, and make connections between and among mathematical ideas and in contexts outside mathematics to build mathematical understanding. As a result, candidates:  4.A Recognize and use connections among mathematical ideas.  4.B Recognize and apply mathematical ideas to other subject areas and to real-world situations.  4.C Demonstrate how mathematical ideas interconnect and build on one another to produce a coherent whole.  5.0 Mathematical Representation Middle level teacher candidates use varied representations (pictorial, verbal, numerical, graphical, or symbolic) of mathematical ideas to support and deepen mathematical understanding in each mathematical content area. As a result, candidates:  5.A Use multiple representations to model and interpret physical, social, and mathematical phenomena.  5.B Create and use representations to organize, record, and communicate mathematical ideas.  5.C Select, translate among, and apply mathematical representations to solve problems. | 8.0 Mathematics Instructional Methodology  Candidates possess a deep understanding of how students learn mathematics and of the pedagogical knowledge specific to mathematics teaching and learning.  8.A Select, use, and determine suitability of the available mathematics curricula, teaching materials, and other resources including manipulatives for the learning of mathematics for all students.  8.B Demonstrate ability to present mathematical concepts using multiple representations (e.g., numerical, graphical, analytical, and contextual).  8.C Demonstrate the ability to guide student discourse in mathematical problem solving, argumentation (creation and critiquing), literacy, and in-depth conceptual understanding.  8.D Demonstrate knowledge of learning progressions, including conceptual and procedural milestones and common misconceptions, within each content domain and connections to instruction.  8.D.1 Demonstrate knowledge of major, supporting, and additional clusters for each grade level.  8.D.2 Demonstrate an understanding of the concept of mathematical rigor including conceptual understanding, procedural skill and fluency, and application.  8.D.3 Demonstrate an understanding of coherent connections within clusters at a grade level and the progression from grade level to grade level that builds on previous learning.  8.E Engage in developmentally and culturally responsive teaching of mathematics that minimizes power and status issues, nurtures a positive mathematics disposition, and utilizes students’ cultural funds of knowledge and experiences as resources for lessons. |
| 11.0 Calculus Middle level teacher candidates demonstrate a conceptual understanding of and procedural facility with fundamental single variable calculus and ways of representing calculus (numbers, tables, graphs, models, words, and symbols). As a result, candidates:  11.A Demonstrate a conceptual and procedural understanding of fundamental elements of calculus including: limit, continuity, differentiation, and integration and a general background in the techniques and application of calculus.  11.B Use technological tools to explore and represent fundamental concepts of calculus.  11.C Demonstrate understanding of the historical development of calculus, including contributions from many cultures.  12.0 Discrete Mathematics: Middle level teacher candidates apply the fundamental ideas of discrete mathematics in the formulation and solution of problems and ways of representing discrete mathematics (numbers, tables, graphs, models, words, and symbols). As a result, candidates:  12.A Demonstrate understanding of fundamentals of discrete mathematics including elements of recurrence relations, finite difference approaches, logic, graph theory (e.g. simple circuits, graphs, trees, and critical path scheduling), linear programming (e.g. basic optimization with small sets of linear functions), and combinatorics (e.g. simple combinations and permeations).  12.B Apply the fundamental ideas of discrete mathematics in the formulation and solution of problems arising from real-world situations.  12.C Use technological tools to solve problems involving the use of discrete structures and the application of algorithms.  12.D Demonstrate understanding of the historical development of discrete mathematics, including contributions from many cultures. |  |