**Side-by-Side Comparison of Current and Proposed Endorsement Competencies with Alignment to CCSS-M & STEM Integration**

**Elementary Education (K-8) Endorsement Competencies Related to Mathematics**

1. **Common Core: Knowledge of Academic Content:** Candidates understand and apply knowledge of the arts, English language arts, health-fitness, mathematics, science, and social studies.

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| **Current Competencies** | **Draft Competencies** |
| 1.D Mathematics | 1.D Mathematics  Candidates possess a deep understanding of development and mathematical and spatial learning from early childhood through adolescence. |
| N/A | 1.D.1 Understand the developmental progression of mathematical learning including, spatial perception, recognition of shapes, visual matching, counting, knowledge of numbers, visual recognition of numbers, ordering, sorting, and classifying, creation of two- and three-dimensional objects, creating and expanding patterns, spatial rotation and number sense. |
| N/A | 1.D.2 Understand and apply the fundamental principles, concepts, and procedures related to mathematical problem solving, exploration, and reasoning, including processes and skills related to using mathematical language to communicate relationships and concepts, adaptive reasoning, strategic competence, procedural fluency, and productive disposition. |
| 1.D.1 Mathematical Problem Solving  1.D.1.A Apply and adapt a variety of appropriate strategies to solve problems of different types.  1.D.1.B Solve problems that arise in mathematics and those involving mathematics in other contexts.  1.D.1.C Build new mathematical knowledge through problem solving.  1.D.1.D Monitor and reflect on the process of mathematical problem solving. | 1.D.3 Standards for Mathematical Practices: Demonstrate ability to embed CCSS-M Mathematical Practices in the instructional process to deepen conceptual understanding.  1.D.3.A Make sense of problems and persevere in solving them.  1.D.3.B Reason abstractly and quantitatively.  1.D.3.C Construct viable arguments and critique the reasoning of others.  1.D.3.D Model with mathematics.  1.D.3.E Use appropriate tools strategically.  1.D.3.F Attend to precision.  1.D.3.G Look for and make use of structure.  1.D.3.H Look for and express regularity in repeated reasoning. |
| 1.D.7 Number and Operation  Develop the meaning of addition, subtraction, multiplication, and division and provide multiple models involving operations with whole numbers, integers, and rational numbers.  1.D.7.A Demonstrate proficiency and flexibility in multi-digit computation using algorithms, mental mathematics, and computational estimation.  1.D.7.B Provide equivalent representations of fractions, decimals, and percents.  1.D.7.C Create, solve, and apply proportions.  1.D.7.D Recognize and apply the fundamental ideas of number theory.  1.D.7.E Make sense of large and small numbers and use scientific notation.  1.D.7.F Analyze and explain the distinctions among whole numbers, integers, rational numbers, and real numbers.  1.D.7.G Recognize the meaning and use of place value in representing whole numbers and finite decimals, comparing and ordering numbers, and understanding the relative magnitude of numbers. | 1.D.4 Counting and Cardinality / Number and Operations in Base Ten & Fractions: Demonstrate a conceptual understanding of and procedural facility and application of operations, number systems, and properties.  1.D.4.A Demonstrate understanding of the progression of learning that begins with the base-ten number system, place value, and operations thereof, builds into understanding of and operations with fractions and rational numbers, and extends to understanding of and operations with real numbers.  1.D.4.B Understand and apply ratios, proportional thinking, and other methods for representing and solving mathematical and real world problems.  1.D.5 Number and Quantity: Understand the progression of learning that begins with the base-ten number system and operations thereof, builds into understanding of and operations with fractions and rational numbers, and extends to understanding of and operations with real numbers.  1.D.5.A Understand the characteristics of and relationships between different number systems including whole numbers, integers, rational, real, and complex numbers.  1.D.5.B Understand arithmetic operations of different number systems and their properties (integers, rational, and irrational numbers). |
| 1.D.8 Multiple Perspectives on Algebra  1.D.8.A Explore, analyze, and represent patterns, relations, and functions.  1.D.8.B Investigate equality, equations, and proportional relationships.  1.D.8.C Use mathematical models to represent quantitative relationships.  1.D.8.D Analyze change in various contexts.  1.D.8.E Demonstrate knowledge of the historical development of algebra, including contributions from many cultures. | 1.D.6 Operations and Algebraic Thinking / Algebra and Functions: Demonstrate a conceptual understanding of and procedural facility with algebra concepts emphasizing functions.  1.D.6.A Understand and apply properties of mathematical operations, strategies for computing and estimating solutions, and methods for modeling mathematical operations.  1.D.6.B Solve and graphically represent real life and mathematical problems using numerical and algebraic expressions, equations, inequalities, and systems of equations and inequalities.  1.D.6.C Understand the connections between proportional relationships, lines, and linear equations and use them to solve real world and mathematical problems.  1.D.6.D Use functional notation and interpret expressions for functions as they arise in terms of the situation they model (e.g., linear, quadratic, simple rational, and exponential).  1.D.6.E Understand operations on algebraic expressions and functions (e.g., polynomials, rationals, and roots).  1.D.6.F Apply arithmetic properties to algebraic expressions and equations.  1.D.6.G Write equations and inequalities in equivalent forms.  1.D.6.H Explain the interrelationship between the various representations of a function (e.g., graphs, tables, algebraic expressions, concrete models, and contexts). |
| 1.D.11 Measurement  1.D.11.A Recognize the common representations and uses of measurement and choose appropriate tools and units for measuring.  1.D.11.B Identify the attributes to be measured and apply appropriate techniques, tools, and formulas to determine measurements and their application in a variety of contexts.  1.D.11.C Use estimation as a way of understanding measurement units and processes.  1.D.11.D Demonstrate knowledge of the historical development of measurement and measurement systems, including contributions from many cultures. | 1.D.7 Measurement and Data: Understand measurement systems and units, concepts related to geometric measurement, and tools and techniques used to solve measurement problems.  1.D.7.A Apply standard units of measurement and estimation.  1.D.7.B Understand processes and skills related to collecting, interpreting, and representing data. |
| 1.D.9 Geometries  1.D.9.A Demonstrate knowledge of core concepts and principles of Euclidean geometry in two and three dimensions.  1.D.9.B Exhibit knowledge of informal proof.  1.D.9.C Build and manipulate representations of two- and three-dimensional objects using concrete models, drawings, and dynamic geometry software, and perceive an object from different perspectives.  1.D.9.D Specify locations and describe spatial relationships using coordinate geometry.  1.D.9.E Analyze properties and relationships of geometric shapes and structures.  1.D.9.F Apply transformations and use symmetry, similarity, and congruence in mathematical situations.  1.D.9.G Demonstrate knowledge of the historical development of Euclidean geometry, including contributions from many cultures. | 1.D.8 Geometry: Demonstrate a conceptual understanding of geometric properties and relationships as they apply to congruence, similarity, geometric figures, and the Cartesian Coordinate System.  1.D.8.A Understand congruence in terms of rigid motion.  1.D.8.B Prove theorems involving triangle congruency and similarity.  1.D.8.C Apply transformations and use similarity and congruence in mathematical situations.  1.D.8.D Understand and perform geometric constructions physically and with technology.  1.D.8.E Understand the Pythagorean Theorem and apply it to problem solving situations.  1.D.8.F Solve real life and mathematical problems involving lines, angle measure, area, surface area, and volume.  1.D.8.G Classify, visualize, and describe two-dimensional figures and three-dimensional objects as well as the relationship among them.  1.D.8.H Apply geometric concepts to model real world situations. |
| 1.D.10 Data Analysis, Statistics, and Probability  1.D.10.A Design investigations, collect data, use a variety of methods to display data, interpret data representations and draw and represent conclusions that may include vicariate data and geometric probability.  1.D.10.B Use appropriate statistical methods and technological tools to analyze data and describe shape, spread, and center.  1.D.10.C Draw conclusions involving uncertainty by using hands-on and technology-based simulation for estimating probabilities and gathering data to make inferences and decisions.  1.D.10.D Identify misuses of statistics and invalid conclusions from probability.  1.D.10.E Demonstrate knowledge of the historical development of probability and statistics, including contributions from many cultures. | 1.D.9 Statistics and Probability: Demonstrate conceptual understanding and procedural facility of statistics and probability.  1.D.9.A Use appropriate measures of central tendency and distributions to summarize, represent, and interpret categorical and quantitative data.  1.D.9.B Understand and evaluate random processes underlying statistical experiments and use random sampling to make inferences about whole populations.  1.D.9.C Understand and use the rules of probability to make predictions, evaluate decisions, and solve problems.  1.D.9.D Apply statistical and probability concepts to model real world situations. |
| N/A | 1.D.10 Ratios and Proportional Relationships: Demonstrate conceptual understanding and procedural fluency in analyzing proportional relationships and solving real world mathematical problems.  1.D.10.A Describe and determine additive versus multiplicative perspectives.  1.D.10.B Reason and compute with ratios and the constant of proportionality (unit rate) to solve real world and mathematical problems.  1.D.10.C Recognize, describe, and represent equivalent ratios, rates, and proportional relationships.  1.D.10.D Represent and analyze proportional relationships using tables, graphs, equations, diagrams, concrete and mathematical models, and verbal descriptions of proportional relationships.  1.D.10.E Compute the constant of proportionality (unit rate) associated with rational numbers.  1.D.10.F Recognize and connect proportional relationships to geometry, measurement, statistics, probability, and function.  1.D.10.G Use ratio reasoning to convert measurement units.  1.D.10.H Apply ratio and proportion concepts to model real world situations. |
| 1.D.6 Technology:  1.D.6.A Use knowledge of mathematics to select and use appropriate technological tools.  1.D.6.B Understand the appropriate use of technology to experiment, visualize, and enable students to make and explore conjectures. | 1.D.11 Modeling and Technology: Connect mathematics with real life problems through the use of mathematical modeling and technology.  1.D.11.A Construct mathematical models in the content strands (e.g., look at a real life situation and transpose it into a mathematical problem, solve the problem, and interpret the solution in real life.)  1.D.11.B Use the appropriate technology available to explore conjectures, visualize, and analyze the mathematics, develop concepts and apply them to a context, and use technology to model in the real world. |
| 1.D.2 Reasoning and Proof  1.D.2.A Recognize reasoning and use of evidence as fundamental aspects of mathematics.  1.D.2.B Make and investigate mathematical conjectures.  1.D.2.C Develop, evaluate and select mathematical arguments and proofs as appropriate  for the K-8 curriculum.  1.D.3 Mathematical Communication  1.D.3.A Systematically gather mathematical information for a given purpose and communicate their mathematical thinking coherently and clearly to peers, faculty, and others.  1.D.3.B Use the language of mathematics to express ideas precisely.  1.D.3.C Organize mathematical thinking through communication.  1.D.3 D Analyze and evaluate the mathematical thinking and strategies of others.  1.D.4 Mathematical Connections  1.D.4.A Recognize and use connections among mathematical ideas.  1.D.4.BRecognize and apply mathematics in real-world contexts.  1.D.4.C Demonstrate how mathematical ideas interconnect and build on one another to produce a coherent whole.  1.D.5 Mathematical Representation  1.D.5.A Use varied representations (words, pictures, data representation) to model and interpret physical, social, and mathematical phenomena.  1.D.5.B Create and use representations to organize, record, and communicate mathematical ideas.  1.D.5.C Select, apply, and translate among mathematical representations to solve problems. | 1.D.12 Mathematics Instructional Methodology: Candidates possess a deep understanding of how students learn mathematics and of the pedagogical knowledge specific to mathematics teaching and learning.  1.D.12.A Select, use, and determine suitability of the available mathematics curricula, teaching materials, and other resources including manipulatives for the learning of mathematics for all students.  1.D.12.B Demonstrate ability to present mathematical concepts using multiple representations (e.g., numerical, graphical, analytical, and contextual).  1.D.12.C Demonstrate the ability to guide student discourse in mathematical problem solving, argumentation (creation and critiquing), literacy, and in-depth conceptual understanding.  1.D.12.D Demonstrate knowledge of learning progressions, including conceptual and procedural milestones and common misconceptions, within each content domain and connections to instruction.  1.D.12.D.1 Demonstrate knowledge of major, supporting, and additional clusters for each grade level.  1.D.12.D.2 Demonstrate an understanding of the concept of mathematical rigor including conceptual understanding, procedural skill and fluency, and application.  1.D.12.D.3 Demonstrate an understanding of coherent connections within clusters at a grade level and the progression from grade level to grade level that builds on previous learning.  1.D.12.E Engage in developmentally and culturally responsive teaching of mathematics that minimizes power and status issues, nurtures a positive mathematics disposition, and utilizes students’ cultural funds of knowledge and experiences as resources for lessons. |